Solving trig equations and the unit circle

Ten trigonometric equations solved exactly on [0, 2π) and in general: sin, cos and tan set to unit-circle values, quadratics in sin x, equations mixing 2x and x, a triple angle, and sin x + cos x = 1, with worked solutions and the mistakes that lose solutions by dividing or gain them by squaring.

BEFORE YOU START

A trigonometric equation usually has two solutions in each turn of the circle, and most mistakes lose one of them or gain one that is not there. The ten problems find every solution on $[0, 2\pi)$ from a handful of exact values and one sign rule, then extend them to every angle.

- x is the unknown angle, in radians. Solve on $[0, 2\pi)$, one full turn, unless the problem asks for the general solution: every solution, of the form "a solution on $[0, 2\pi)$ plus a whole number of periods". In a general solution, k stands for any integer, $k \in \mathbb{Z}$.
- On the unit circle, the point at angle x is $(\cos x, \sin x)$: cosine is the horizontal coordinate and sine the vertical one. The exact values needed on this page:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

- Sine is positive in quadrants I and II, cosine in I and IV, tangent in I and III. A reference angle θ in quadrant I, read from the table, gives one solution; the quadrant rule gives the other, at $\pi - \theta$ (quadrant II), $\pi + \theta$ (III) or $2\pi - \theta$ (IV). Each of these has the same sine and cosine as θ up to sign.
- Identities used: $\sin^2 x + \cos^2 x = 1$, $\sin 2x = 2 \sin x \cos x$, $\cos 2x = 2\cos^2 x - 1$ and the sum formula $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$, all from the Trig identities page.

Builds on: Trig identities

Problems

1. Solve $\sin x = \frac{1}{2}$ on $[0, 2\pi)$, then give the general solution. difficulty ·

2. Solve $\cos x = -\dfrac{\sqrt{3}}{2}$ on $[0, 2\pi)$. difficulty ·

3. Solve $2\sin^2 x - \sin x - 1 = 0$ on $[0, 2\pi)$. difficulty ··

4. Solve $\sin 2x = \cos x$ on $[0, 2\pi)$. difficulty ··

5. Solve $\tan x = \sqrt{3}$ on $[0, 2\pi)$, then give the general solution. difficulty ··

6. Solve $\cos 2x = \cos x$ on $[0, 2\pi)$. difficulty ···

7. Solve $\sin x = \cos x$ on $[0, 2\pi)$ two ways. difficulty ···

8. Solve $2\cos 3x = 1$ on $[0, 2\pi)$. How many solutions should there be, before you find them? difficulty ··

9. Solve $\sin x + \cos x = 1$ on $[0, 2\pi)$. Show what goes wrong if you square both sides. difficulty ···

10. Give the general solution of $\sin^2 x = \frac{3}{4}$. difficulty ···

Answers

1. $x = \frac{\pi}{6}, \frac{5\pi}{6}$; general $x = \frac{\pi}{6} + 2k\pi$ or $x = \frac{5\pi}{6} + 2k\pi$
2. $x = \frac{5\pi}{6}, \frac{7\pi}{6}$
3. $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$
4. $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$
5. $x = \frac{\pi}{3}, \frac{4\pi}{3}$; general $x = \frac{\pi}{3} + k\pi$
6. $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$
7. $x = \frac{\pi}{4}, \frac{5\pi}{4}$
8. $x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$
9. $x = 0, \frac{\pi}{2}$; squaring gives $1 + \sin 2x = 1$, i.e. $\sin 2x = 0$, whose solutions $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ include two extraneous ones (π and $\frac{3\pi}{2}$ give -1)
10. $x = \pm\frac{\pi}{3} + k\pi$ (on $[0, 2\pi)$: $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$)

Worked solutions

Problem 1

> Solve $\sin x = \frac{1}{2}$ on $[0, 2\pi)$, then give the general solution.

$\sin \frac{\pi}{6} = \frac{1}{2}$, so the reference angle is $\frac{\pi}{6}$.	From the table: $\frac{\pi}{6}$ is the quadrant-I angle whose sine is $\frac{1}{2}$.
$x = \frac{\pi}{6}$ or $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.	$\frac{1}{2} > 0$, and sine is positive in quadrants I and II only, so there is one solution in each. The quadrant-II angle $\pi - \frac{\pi}{6}$ is the mirror image of $\frac{\pi}{6}$ across the vertical axis, at the same height, so it has the same sine.
Every solution is $\frac{\pi}{6}$ or $\frac{5\pi}{6}$ plus a whole number of turns.	Sine has period 2π: adding a full turn returns to the same point of the circle, and $[0, 2\pi)$ is exactly one turn, so every angle is one of its points plus $2k\pi$.
$x = \frac{\pi}{6}, \frac{5\pi}{6}$; **general** $x = \frac{\pi}{6} + 2k\pi$ **or** $x = \frac{5\pi}{6} + 2k\pi$	Steps 2 and 3.

Problem 2

> Solve $\cos x = -\dfrac{\sqrt{3}}{2}$ on $[0, 2\pi)$.

$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, so the reference angle is $\frac{\pi}{6}$.	The table gives the value without its sign; the quadrant rule supplies the sign.
The solutions lie in quadrants II and III.	$-\frac{\sqrt{3}}{2} < 0$, and cosine is positive in quadrants I and IV, so it is negative in II and III.
$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ and $x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$.	These are the quadrant-II and quadrant-III angles with reference angle $\frac{\pi}{6}$. Both lie at horizontal coordinate $-\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$.

$x = \frac{5\pi}{6}, \frac{7\pi}{6}$	Step 3.

Problem 3

Solve $2\sin^2 x - \sin x - 1 = 0$ on $[0, 2\pi)$.

Let $s = \sin x$; the equation is $2s^2 - s - 1 = 0$.	Only $\sin x$ appears, so this is a quadratic in the one number s.
$2s^2 - s - 1 = (2s + 1)(s - 1)$.	Expanding the right side gives $2s^2 - 2s + s - 1$, which is the left side.
$\sin x = 1$ or $\sin x = -\frac{1}{2}$.	A product is zero exactly when one of its factors is.
$\sin x = 1$ gives $x = \frac{\pi}{2}$ only.	The unit circle reaches height 1 at one point, the top, $(0, 1)$. Its quadrant-II partner $\pi - \frac{\pi}{2}$ is $\frac{\pi}{2}$ again, so this value has one solution per turn, not two.
$\sin x = -\frac{1}{2}$ gives $x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ and $x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$.	The reference angle is $\frac{\pi}{6}$ (Problem 1), and $-\frac{1}{2} < 0$, so the solutions lie in quadrants III and IV, where sine is negative.
$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$	Steps 4 and 5 together, in increasing order.

Problem 4

Solve $\sin 2x = \cos x$ on $[0, 2\pi)$.

$2\sin x \cos x = \cos x$.	The double-angle formula writes both sides in terms of the same angle x.

$2\sin x\cos x - \cos x = \cos x\,(2\sin x - 1) = 0$.	Move everything to one side and factor out $\cos x$. Dividing both sides by $\cos x$ instead would assume $\cos x \neq 0$ and throw away every solution where $\cos x = 0$ (mistake 1); as a factor, $\cos x = 0$ stays one of the cases.
$\cos x = 0$ gives $x = \frac{\pi}{2}, \frac{3\pi}{2}$.	Cosine is the horizontal coordinate, which is 0 at the top and the bottom of the circle.
$2\sin x - 1 = 0$, that is $\sin x = \frac{1}{2}$, gives $x = \frac{\pi}{6}, \frac{5\pi}{6}$.	Problem 1.
$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$	The two cases together, in increasing order. At $\frac{\pi}{2}$: $\sin\pi = 0 = \cos\frac{\pi}{2}$.

Problem 5

> Solve $\tan x = \sqrt{3}$ on $[0, 2\pi)$, then give the general solution.

$\tan\frac{\pi}{3} = \dfrac{\sin(\pi/3)}{\cos(\pi/3)} = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}$, so the reference angle is $\frac{\pi}{3}$.	$\tan x = \sin x / \cos x$, with both values from the table.
$x = \frac{\pi}{3}$ or $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$.	$\sqrt{3} > 0$, and tangent is positive in quadrants I and III, where sine and cosine have the same sign.
$\tan(x + \pi) = \tan x$ wherever $\tan x$ is defined.	Adding π moves $(\cos x, \sin x)$ to the opposite point $(-\cos x, -\sin x)$, and negating both leaves their ratio unchanged. So tangent has period π, and the two solutions in step 2 are one solution and its shift by π.
$x = \frac{\pi}{3}, \frac{4\pi}{3}$; **general** $x = \frac{\pi}{3} + k\pi$	Every solution is $\frac{\pi}{3}$ plus a whole number of periods; even k gives $\frac{\pi}{3}$ plus whole turns, odd k gives $\frac{4\pi}{3}$ plus whole turns.

Problem 6

> Solve $\cos 2x = \cos x$ on $[0, 2\pi)$.

$2\cos^2 x - 1 = \cos x.$	Of the three forms of $\cos 2x$, $2\cos^2 x - 1$ is the one with only cosines, so the equation becomes one in $\cos x$ alone.
$2\cos^2 x - \cos x - 1 = (2\cos x + 1)(\cos x - 1) = 0.$	Move everything to one side and factor the quadratic in $\cos x$; expanding gives $2\cos^2 x - 2\cos x + \cos x - 1$. Factoring finds every solution, because a product is zero exactly when one factor is; nothing is divided, so no case is lost.
$\cos x = 1$ gives $x = 0.$	The circle's horizontal coordinate is 1 at one point only, $(1, 0)$, which is the angle 0; 2π is the same point but is outside $[0, 2\pi)$.
$\cos x = -\frac{1}{2}$ gives $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ and $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}.$	$\cos \frac{\pi}{3} = \frac{1}{2}$ gives the reference angle, and $-\frac{1}{2} < 0$ puts the solutions in quadrants II and III, where cosine is negative.
$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$	Steps 3 and 4, in increasing order. At $\frac{2\pi}{3}$: $\cos \frac{4\pi}{3} = -\frac{1}{2} = \cos \frac{2\pi}{3}$.

Problem 7

> Solve $\sin x = \cos x$ on $[0, 2\pi)$ two ways.

At any solution, $\cos x \neq 0.$	If $\cos x = 0$, the equation would force $\sin x = 0$ as well, but $\sin^2 x + \cos^2 x = 1$ rules out both being 0. So dividing by $\cos x$ loses nothing here, unlike Problem 4, where $\cos x = 0$ did give solutions.
$\tan x = 1.$	Divide both sides by $\cos x$, nonzero at every solution by step 1.
$x = \frac{\pi}{4}$ or $x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}.$	$\sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$, so $\tan \frac{\pi}{4} = 1$ and $\frac{\pi}{4}$ is the reference angle; $1 > 0$, and tangent is positive in quadrants I and III.

$x = \frac{\pi}{4}, \frac{5\pi}{4}$	Step 3.

Another route: as a single sine

$\sin x - \cos x = 0.$	Move $\cos x$ to the left, so that the left side can be written as one sine.
$\sin x - \cos x = \sqrt{2}\big(\sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4}\big) = \sqrt{2}\sin(x - \frac{\pi}{4}).$	$\sqrt{2}\cos\frac{\pi}{4} = \sqrt{2}\sin\frac{\pi}{4} = \sqrt{2}\cdot\frac{\sqrt{2}}{2} = 1$, so the bracket times $\sqrt{2}$ is the left side; the bracket is the sum formula with the lower sign.
$\sin(x - \frac{\pi}{4}) = 0$, so $x - \frac{\pi}{4} = 0$ or π.	Divide by $\sqrt{2}$. Sine is 0 at the angles $k\pi$, and as x runs over $[0, 2\pi)$, $x - \frac{\pi}{4}$ runs over $[-\frac{\pi}{4}, \frac{7\pi}{4})$, which contains 0 and π only.
$x = \frac{\pi}{4}, \frac{5\pi}{4}$	Add $\frac{\pi}{4}$. The two routes agree.

Problem 8

> Solve $2\cos 3x = 1$ on $[0, 2\pi)$. How many solutions should there be, before you find them?

As x runs over $[0, 2\pi)$, $3x$ runs over $[0, 6\pi)$: three full turns.	Multiplying by 3 stretches the interval to three times its length.
There are six solutions.	$\cos 3x = \frac{1}{2}$, and cosine takes the value $\frac{1}{2}$ twice in each turn (once in quadrant I and once in IV, since $\frac{1}{2}$ is strictly between -1 and 1), so three turns give $3 \times 2 = 6$.
$\cos 3x = \frac{1}{2}$, so $3x = \pm\frac{\pi}{3} + 2k\pi$.	Divide by 2. $\cos\frac{\pi}{3} = \frac{1}{2}$ gives the reference angle; cosine is positive in quadrants I and IV, and the quadrant-IV solution $2\pi - \frac{\pi}{3}$ is $-\frac{\pi}{3}$ plus a turn. Cosine has period 2π.

$3x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}$.	The values of step 3 in $[0, 6\pi)$: $\frac{\pi}{3} + 2k\pi$ for $k = 0, 1, 2$ and $-\frac{\pi}{3} + 2k\pi$ for $k = 1, 2, 3$.
$x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$	Divide by 3. Six solutions, as step 2 predicted.

Problem 9

> Solve $\sin x + \cos x = 1$ on $[0, 2\pi)$. Show what goes wrong if you square both sides.

$\sin x + \cos x = \sqrt{2}\big(\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}\big) = \sqrt{2}\sin(x + \frac{\pi}{4})$.	$\sqrt{2}\cos\frac{\pi}{4} = \sqrt{2}\sin\frac{\pi}{4} = 1$, and the bracket is the sum formula with the upper sign. One sine is easier to solve than a sum of two.
$\sin(x + \frac{\pi}{4}) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.	Divide the equation $\sqrt{2}\sin(x + \frac{\pi}{4}) = 1$ by $\sqrt{2}$.
$x + \frac{\pi}{4} = \frac{\pi}{4}$ or $\frac{3\pi}{4}$.	The reference angle is $\frac{\pi}{4}$ and sine is positive in quadrants I and II, giving $\frac{\pi}{4}$ and $\pi - \frac{\pi}{4}$. As x runs over $[0, 2\pi)$, $x + \frac{\pi}{4}$ runs over $[\frac{\pi}{4}, \frac{9\pi}{4})$; the next solution, $\frac{\pi}{4} + 2\pi$, is just outside it.
$x = 0$ or $x = \frac{\pi}{2}$.	Subtract $\frac{\pi}{4}$. At 0: $0 + 1 = 1$; at $\frac{\pi}{2}$: $1 + 0 = 1$.
Squaring instead: $\sin^2 x + 2\sin x \cos x + \cos^2 x = 1$, so $1 + \sin 2x = 1$ and $\sin 2x = 0$.	Expand the square; $\sin^2 x + \cos^2 x = 1$ and $2\sin x \cos x = \sin 2x$.
$2x = 0, \pi, 2\pi, 3\pi$, so the candidates are $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$.	Sine is 0 at the multiples of π, and $2x$ runs over $[0, 4\pi)$.

At π: $\sin\pi + \cos\pi = 0 - 1 = -1$. At $\frac{3\pi}{2}$: $-1 + 0 = -1$.	Substitute each candidate into the original equation. Squaring cannot tell 1 from -1, since both square to 1, so the squared equation also holds wherever $\sin x + \cos x = -1$; these two candidates are those points.
$x = 0, \frac{\pi}{2}$; **squaring gives** $1 + \sin 2x = 1$, **i.e.** $\sin 2x = 0$, **whose solutions** $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ **include two extraneous ones (π and $\frac{3\pi}{2}$ give -1)**	Steps 4 and 7: of the four candidates, only 0 and $\frac{\pi}{2}$ satisfy the original equation, as step 4 found without squaring.

Problem 10

> Give the general solution of $\sin^2 x = \frac{3}{4}$.

$\sin x = \frac{\sqrt{3}}{2}$ or $\sin x = -\frac{\sqrt{3}}{2}$.	The numbers whose square is $\frac{3}{4}$ are $\pm\frac{\sqrt{3}}{2}$. Keeping both signs loses nothing and adds nothing, so unlike squaring in Problem 9 this step is reversible.
On $[0, 2\pi)$: $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$.	$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ gives the reference angle. The plus sign gives quadrants I and II ($\frac{\pi}{3}$, $\pi - \frac{\pi}{3}$), the minus sign quadrants III and IV ($\pi + \frac{\pi}{3}$, $2\pi - \frac{\pi}{3}$): one solution in every quadrant.
$\sin^2(x + \pi) = \sin^2 x$.	$\sin(x + \pi) = -\sin x$, because adding π moves to the opposite point of the circle, and squaring removes the sign. So $\sin^2 x$ has period π, not 2π.
$\frac{4\pi}{3} = \frac{\pi}{3} + \pi$, $\frac{2\pi}{3} = -\frac{\pi}{3} + \pi$ and $\frac{5\pi}{3} = -\frac{\pi}{3} + 2\pi$.	By step 3, the four solutions of step 2 are $\frac{\pi}{3}$ and $-\frac{\pi}{3}$ shifted by whole periods π.
$x = \pm\frac{\pi}{3} + k\pi$ **(on** $[0, 2\pi)$**:** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$**)**	Steps 3 and 4: every solution is $\frac{\pi}{3}$ or $-\frac{\pi}{3}$ plus a whole number of periods.

Where this goes wrong

1. Dividing by cos x and losing two solutions

Canceling a factor that appears on both sides is a reflex from ordinary algebra.

$\sin 2x = \cos x$, that is $2 \sin x \cos x = \cos x$	Right so far: Problem 4 with the double-angle formula applied.
"$\cos x$ is on both sides, so cancel it."	The habit that causes the mistake: canceling as in $3a = 3 \Rightarrow a = 1$, where the factor is a number known not to be 0.
$\sin 2x = \cos x \Rightarrow 2 \sin x = 1$ ✗	Dividing by $\cos x$ assumes $\cos x \neq 0$, so it discards the solutions where $\cos x = 0$, which are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$; both solve the equation ($\sin \pi = 0 = \cos \frac{\pi}{2}$). This line keeps only $\frac{\pi}{6}$ and $\frac{5\pi}{6}$. Factor instead: $\cos x \, (2 \sin x - 1) = 0$ keeps all four (Problem 4).

2. Keeping the extraneous roots after squaring

Squaring both sides is a legitimate move, so its result looks like the answer.

$(\sin x + \cos x)^2 = 1$, so $\sin 2x = 0$ and $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$	Right so far: every solution of $\sin x + \cos x = 1$ is among these four (Problem 9).
"Squaring both sides keeps the equation true, so its solutions are the answer."	The half-truth that causes the mistake: squaring keeps every solution, but it can add new ones.
$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ for $\sin x + \cos x = 1$ ✗	Squaring is not reversible: $a^2 = 1$ allows $a = -1$ too. At π and $\frac{3\pi}{2}$, $\sin x + \cos x = -1$, so they satisfy the squared equation only. Substitute every candidate back into the original equation; the solutions are 0 and $\frac{\pi}{2}$.

3. Only the reference angle

A calculator's inverse sine returns one angle, and it is tempting to stop there.

$\sin x = \frac{1}{2}$ on $[0, 2\pi)$	Right so far: Problem 1.
"$\sin \frac{\pi}{6} = \frac{1}{2}$, so $x = \frac{\pi}{6}$."	The habit that causes the mistake: $\arcsin \frac{1}{2} = \frac{\pi}{6}$ is one solution, the one in $[-\frac{\pi}{2}, \frac{\pi}{2}]$, not all of them.
$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$ only ✗	Sine is positive in quadrant II too; $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$ is the other solution, the mirror image of $\frac{\pi}{6}$ across the vertical axis, at the same height.

4. Solving for 3x on [0, 2π) instead of [0, 6π)

The interval $[0, 2\pi)$ is so familiar that it gets applied to whatever sits inside the cosine.

$2\cos 3x = 1$, so $\cos 3x = \frac{1}{2}$, with x in $[0, 2\pi)$	Right so far: Problem 8.
"Cosine is $\frac{1}{2}$ at $\frac{\pi}{3}$ and $\frac{5\pi}{3}$, so $3x$ is one of those."	The habit that causes the mistake: solving for the angle $3x$ on $[0, 2\pi)$ without asking what interval $3x$ covers.
$2\cos 3x = 1 \Rightarrow x = \frac{\pi}{9}, \frac{5\pi}{9}$ only ✗	$3x$ covers three full periods, $[0, 6\pi)$, while x covers one; the count is $3 \times 2 = 6$. The missing four have $3x$ past the first turn: at $x = \frac{7\pi}{9}$, $\cos \frac{7\pi}{3} = \cos \frac{\pi}{3} = \frac{1}{2}$.

5. The wrong period in the general solution of tan

Sine and cosine repeat every 2π, and that period gets carried over to tangent.

$\tan x = \sqrt{3}$, with the solution $x = \frac{\pi}{3}$	Right so far: Problem 5.
"Trig functions repeat every 2π, so add $2k\pi$."	The habit that causes the mistake: a fact about sine and cosine applied to tangent.

$x = \frac{\pi}{3} + 2k\pi$ for $\tan x = \sqrt{3}$ ✗	The period of tangent is π, not 2π: adding π negates both sine and cosine, leaving their ratio unchanged. The $2k\pi$ version misses $\frac{4\pi}{3}$ and every $\frac{\pi}{3} + k\pi$ with k odd (Problem 5).

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