Graphs of sine and cosine: amplitude, period and phase shift

Ten problems on the graphs of sine and cosine: amplitude, period, midline and range read off a formula, the phase shift of sin(2x − π/3) with B factored out, the five key points of one period, cosine as a shifted sine, an equation built from a maximum and the next minimum, a daylight model and the days above 14 hours, hertz against radians per second for a mains voltage, the period of sin 2x + cos 3x, why the order of a stretch and a shift matters, and the wavelengths of a transformer's positional encoding with the rotation that moves it k positions, with worked solutions and the mistakes of reading the phase shift before factoring out B, taking the period as 2πB, taking the maximum as the amplitude, dropping the midline's sign and evaluating in degrees.

BEFORE YOU START

Every sine wave is the graph of $\sin x$ stretched, shifted and flipped, and four numbers say exactly how: the amplitude, the period, the phase shift and the midline. Reading them off a formula, and writing the formula from a graph or a description, is the skill behind models of daylight, tides, alternating current and sound, and behind the sinusoidal positional encodings of a transformer. These ten problems read the four numbers off formulas, sketch one period from five key points, turn a cosine into a sine, build an equation from a maximum and a minimum, use a daylight model, convert between hertz and radians per second, find the period of a sum, and end with the positional encoding and the rotation hidden in it. The five mistakes at the end are the ones that still give a believable graph: a phase shift read before factoring out B, a period of $2\pi B$, the maximum taken as the amplitude, the midline's sign dropped, and a calculator left in degrees.

- **The general sinusoid.** $y = A\sin\big(B(x - C)\big) + D$, and the same with cos, with $A \neq 0$ and $B \neq 0$. Angles are in radians throughout.
- **Amplitude** $|A|$: half the distance from the minimum to the maximum. A negative A flips the graph across the midline.
- **Period** $2\pi/|B|$: the length of one full cycle. $|B|$ is the **angular frequency**, in radians per unit of x, and $|B|/(2\pi)$, cycles per unit of x, is the **frequency**.
- **Phase shift** C: the graph of $A\sin(Bx) + D$ moved C units to the right ($C < 0$ moves it left). It is read only after B has been factored out of the argument.
- **Midline** $y = D$: the level halfway between the maximum $D + |A|$ and the minimum $D - |A|$.

- One period of $\sin x$, from 0 to 2π, has five **key points**: midline at 0, maximum at $\frac{\pi}{2}$, midline at π, minimum at $\frac{3\pi}{2}$, midline at 2π. For $\cos x$ the list is maximum, midline, minimum, midline, maximum.
- A function f is **periodic** with period $T > 0$ if $f(x + T) = f(x)$ for every x; "the" period is the smallest such T.

Problems

1. Find the amplitude, period, midline and range of $y = 3\sin(4x) - 1$ and of $y = -2\cos(x/3) + 5$. difficulty ·

2. For $y = 2\sin(2x - \pi/3) + 1$, find the amplitude, period and phase shift, and the first $x \geq 0$ where y is largest. difficulty ··

3. List the five key points of one period of $y = 3\sin\left(\frac{x}{2} - \frac{\pi}{4}\right) + 2$, starting at the phase shift. difficulty ··

4. Show that $\cos x = \sin\left(x + \frac{\pi}{2}\right)$. Then write $y = 5\cos(3x)$ in the form $A\sin(B(x - C))$ with $A > 0$, and write $y = -2\sin x$ as a cosine with a positive amplitude. difficulty ··

5. A sinusoid has a maximum of 7 at $x = 1$, and its next minimum is -1, at $x = 4$. Write it as $y = A\cos(B(x - C)) + D$ with $A, B > 0$, and then as a sine. difficulty ··

6. A model of the hours of daylight on day t of the year at a northern latitude is difficulty ···

$$H(t) = 12 + 3\sin\!\left(\frac{2\pi}{365}(t - 80)\right).$$

Give its period, midline and amplitude, and the daylight on day 172. On which days of the year does it give more than 14 hours?

7. A mains voltage is $V(t) = 170\sin(120\pi t)$ volts, with t in seconds. Find its angular frequency, frequency, period and amplitude, and its root-mean-square value $\sqrt{\text{mean of } V^2}$ over one period. difficulty ·

8. Find the period of $f(x) = \sin(2x) + \cos(3x)$. Is $g(x) = \sin x + \sin(\sqrt{2}\,x)$ periodic? difficulty ···

9. Start from $y = \sin x$. (a) Stretch the graph horizontally by a factor of 4, then shift it π to the right. (b) Shift it π to the right, then stretch it horizontally by a factor of 4. Write each equation and give its phase shift. difficulty ··

10. A transformer's sinusoidal positional encoding gives position p the features difficulty ···

$$\begin{aligned} \mathrm{PE}(p, 2i) &= \sin(\omega_i p), \\ \mathrm{PE}(p, 2i+1) &= \cos(\omega_i p), \end{aligned}$$

with $\omega_i = 10000^{-2i/d}$, for $i = 0, \dots, d/2 - 1$. With $d = 512$, find the period in positions of pair $i = 0$, $i = 128$ and $i = 255$. Then show that for each pair there is a 2×2 matrix R_k, depending on k but not on p, that sends the pair at position p to the pair at position $p + k$.

Answers

1. $3\sin(4x) - 1$: amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, range $[-4, 2]$. $-2\cos(x/3) + 5$: amplitude 2, period 6π, midline $y = 5$, range $[3, 7]$
2. Amplitude 2, period π, phase shift $\frac{\pi}{6}$ to the right; the first maximum for $x \geq 0$ is at $x = \frac{5\pi}{12} \approx 1.309$, where $y = 3$
3. $(\frac{\pi}{2}, 2)$, $(\frac{3\pi}{2}, 5)$, $(\frac{5\pi}{2}, 2)$, $(\frac{7\pi}{2}, -1)$, $(\frac{9\pi}{2}, 2)$
4. $5\cos(3x) = 5\sin\left(3(x + \frac{\pi}{6})\right)$, phase shift $-\frac{\pi}{6}$; $-2\sin x = 2\cos\left(x + \frac{\pi}{2}\right)$
5. $y = 4\cos\left(\frac{\pi}{3}(x - 1)\right) + 3 = 4\sin\left(\frac{\pi}{3}(x + \frac{1}{2})\right) + 3$
6. Period 365 days, midline 12 hours, amplitude 3 hours; $H(172) \approx 15.00$ hours; more than 14 hours from day 123 to day 220
7. $120\pi \approx 377$ rad/s, 60 Hz, period $\frac{1}{60}$ s, amplitude 170 V, RMS $170/\sqrt{2} \approx 120.2$ V
8. f has period 2π; g is not periodic
9. (a) $y = \sin\left(\frac{1}{4}(x - \pi)\right)$, phase shift π; (b) $y = \sin\left(\frac{1}{4}(x - 4\pi)\right)$, phase shift 4π; both have period 8π
10. Periods $2\pi \approx 6.28$, $200\pi \approx 628.3$ and about 60611 positions; $R_k = \begin{bmatrix} \cos\omega_i k & \sin\omega_i k \\ -\sin\omega_i k & \cos\omega_i k \end{bmatrix}$ sends $(\sin\omega_i p, \cos\omega_i p)$ to $(\sin\omega_i(p+k), \cos\omega_i(p+k))$

Worked solutions

Problem 1

> Find the amplitude, period, midline and range of $y = 3\sin(4x) - 1$ and of $y = -2\cos(x/3) + 5$.

For $3\sin(4x) - 1$: $A = 3$, $B = 4$, $D = -1$	Match it to $A\sin(B(x - C)) + D$ with $C = 0$.
Amplitude 3, period $2\pi/4 = \frac{\pi}{2}$, midline $y = -1$	The definitions.
Range $[-1 - 3, -1 + 3] = [-4, 2]$	Maximum $D + \lvert A\rvert$, minimum $D - \lvert A\rvert$.
For $-2\cos(x/3) + 5$: $A = -2$, $B = \frac{1}{3}$, $D = 5$	$x/3 = \frac{1}{3}x$.
Amplitude $\lvert -2\rvert = 2$, period $2\pi/\frac{1}{3} = 6\pi$, midline $y = 5$	A small B means a long period.
$3\sin(4x) - 1$: **amplitude** 3, **period** $\frac{\pi}{2}$, **midline** $y = -1$, **range** $[-4, 2]$. $-2\cos(x/3) + 5$: **amplitude** 2, **period** 6π, **midline** $y = 5$, **range** $[3, 7]$	The negative A does not shrink anything: it starts the cosine at its minimum, 3 at $x = 0$, instead of its maximum.

Problem 2

> For $y = 2\sin(2x - \pi/3) + 1$, find the amplitude, period and phase shift, and the first $x \geq 0$ where y is largest.

$2x - \frac{\pi}{3} = 2\left(x - \frac{\pi}{6}\right)$	Factor $B = 2$ out of the whole argument.
Amplitude 2, period $2\pi/2 = \pi$, phase shift $\frac{\pi}{6}$	Now it matches $A\sin(B(x - C)) + D$ with $C = \frac{\pi}{6}$.

The maximum is where $2\left(x-\frac{\pi}{6}\right)=\frac{\pi}{2}+2\pi k$	sin peaks at $\frac{\pi}{2}$ plus whole periods.
$x=\frac{\pi}{6}+\frac{\pi}{4}+\pi k=\frac{5\pi}{12}+\pi k$	Divide by 2, then add $\frac{\pi}{6}$.
Amplitude 2, period π, phase shift $\frac{\pi}{6}$ to the right; the first maximum for $x\geq 0$ is at $x=\frac{5\pi}{12}\approx 1.309$, where $y=3$	$k=-1$ gives $\frac{5\pi}{12}-\pi<0$. Equivalently, the peak of $\sin(2x)$ at $\frac{\pi}{4}$ moved right by $\frac{\pi}{6}$.

Problem 3

> List the five key points of one period of $y=3\sin\left(\frac{x}{2}-\frac{\pi}{4}\right)+2$, starting at the phase shift.

$\frac{x}{2}-\frac{\pi}{4}=\frac{1}{2}\left(x-\frac{\pi}{2}\right)$	Factor out $B=\frac{1}{2}$: phase shift $\frac{\pi}{2}$.
Period $2\pi/\frac{1}{2}=4\pi$, so the key points are π apart	Five points split one period into four equal quarters.
$x=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2},\frac{9\pi}{2}$	Start at the phase shift and add a quarter period each time.
$y=2,5,2,-1,2$	Midline 2, then $2+3$, midline, $2-3$, midline: the pattern of sin.
$(\frac{\pi}{2},2),(\frac{3\pi}{2},5),(\frac{5\pi}{2},2),(\frac{7\pi}{2},-1),(\frac{9\pi}{2},2)$	At $x=\frac{\pi}{2}$ the argument is 0, the start of a sine cycle, so the graph crosses the midline going up. Join the points with the smooth sine shape and repeat every 4π.

Problem 4

> Show that $\cos x = \sin\left(x + \frac{\pi}{2}\right)$. Then write $y = 5\cos(3x)$ in the form $A\sin(B(x - C))$ with $A > 0$, and write $y = -2\sin x$ as a cosine with a positive amplitude.

$\sin\left(x + \frac{\pi}{2}\right) = \sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2} = \cos x$	The sum formula, with $\cos\frac{\pi}{2} = 0$ and $\sin\frac{\pi}{2} = 1$.
$5\cos(3x) = 5\sin\left(3x + \frac{\pi}{2}\right)$	Step 1 with $3x$ in place of x.
$3x + \frac{\pi}{2} = 3\left(x + \frac{\pi}{6}\right)$	Factor out $B = 3$: $C = -\frac{\pi}{6}$.
$\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2} = -\sin x$	The cosine sum formula.
$5\cos(3x) = 5\sin\left(3\left(x + \frac{\pi}{6}\right)\right)$, **phase shift** $-\frac{\pi}{6}$; $-2\sin x = 2\cos\left(x + \frac{\pi}{2}\right)$	A cosine is a sine a quarter period earlier: here the period is $\frac{2\pi}{3}$, and a quarter of it is $\frac{\pi}{6}$. Any sinusoid can be written with either function; only C changes. Combining a sine and a cosine of the same B into one wave is the $a\sin x + b\cos x$ identity of the trig identities page.

Problem 5

> A sinusoid has a maximum of 7 at $x = 1$, and its next minimum is -1, at $x = 4$. Write it as $y = A\cos(B(x - C)) + D$ with $A, B > 0$, and then as a sine.

$A = \frac{7-(-1)}{2} = 4$, $D = \frac{7+(-1)}{2} = 3$	Half the distance from minimum to maximum, and the level halfway between them.
A maximum to the next minimum is half a period: $4 - 1 = 3$, so the period is 6	A cycle goes maximum, minimum, maximum.
$B = 2\pi/6 = \frac{\pi}{3}$	Period $= 2\pi/B$.

$C = 1$	cos peaks where its argument is 0, so put that peak at $x = 1$.
As a sine, the peak sits a quarter period, $\frac{6}{4} = 1.5$, after the start of the cycle: $C = 1 - 1.5 = -\frac{1}{2}$	sin peaks a quarter period after its midline crossing.
$y = 4\cos\left(\frac{\pi}{3}(x-1)\right) + 3 = 4\sin\left(\frac{\pi}{3}(x+\frac{1}{2})\right) + 3$	Check: at $x = 1$ the cosine is $\cos 0 = 1$, so $y = 7$; at $x = 4$ it is $\cos\pi = -1$, so $y = -1$. In the sine form the arguments are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, with the same values.

Problem 6

A model of the hours of daylight on day t of the year at a northern latitude is

$$H(t) = 12 + 3\sin\Big(\frac{2\pi}{365}(t-80)\Big).$$

Give its period, midline and amplitude, and the daylight on day 172. On which days of the year does it give more than 14 hours?

Period $2\pi/\frac{2\pi}{365} = 365$ days, midline 12 hours, amplitude 3 hours	The model repeats every year, between 9 and 15 hours.
At $t = 172$: the argument is $\frac{2\pi}{365}\cdot 92 \approx 1.5837$	Close to $\frac{\pi}{2} \approx 1.5708$.
$H(172) \approx 12 + 3(0.99992) \approx 15.00$	Day 172 is about June 21, the longest day: the peak is at $t = 80 + \frac{365}{4} = 171.25$.
$H(t) > 14$ when $\sin\theta > \frac{2}{3}$, with $\theta = \frac{2\pi}{365}(t-80)$	Subtract 12, divide by 3.
$\arcsin\frac{2}{3} \approx 0.7297 < \theta < \pi - 0.7297 \approx 2.4119$	On one cycle the sine is above $\frac{2}{3}$ between the two angles where it equals $\frac{2}{3}$.

$t - 80 = \frac{365}{2\pi}\theta$, so $122.39 < t < 220.11$	Undo the scaling and the shift.
Period 365 **days, midline** 12 **hours, amplitude** 3 **hours;** $H(172) \approx 15.00$ **hours; more than** 14 **hours from day** 123 **to day** 220	That is 98 whole days, from a stretch of about 97.7 days: $H(122) \approx 13.98$ and $H(123) \approx 14.02$, $H(220) \approx 14.004$ and $H(221) \approx 13.97$.

Problem 7

> A mains voltage is $V(t) = 170\sin(120\pi t)$ volts, with t in seconds. Find its angular frequency, frequency, period and amplitude, and its root-mean-square value $\sqrt{\text{mean of } V^2}$ over one period.

Angular frequency $B = 120\pi \approx 376.99$ radians per second	The coefficient of t.
Frequency $B/(2\pi) = 60$ cycles per second, 60 Hz	One cycle is 2π radians.
Period $1/60 \approx 0.01667$ s	Period $= 2\pi/B$, the reciprocal of the frequency.
Mean of $\sin^2$ over a period is $\frac{1}{2}$	$\sin^2 u = \frac{1}{2}(1 - \cos 2u)$, and $\cos 2u$ averages to 0 over whole cycles.
$120\pi \approx 377$ **rad/s,** 60 **Hz, period** $\frac{1}{60}$ **s, amplitude** 170 **V, RMS** $170/\sqrt{2} \approx 120.2$ **V**	The "120 volts" on a North American outlet is this RMS value; the peak is 170. Hertz count cycles, radians per second count angle, and they differ by 2π.

Problem 8

> Find the period of $f(x) = \sin(2x) + \cos(3x)$. Is $g(x) = \sin x + \sin(\sqrt{2}\,x)$ periodic?

$\sin(2x)$ has period π and $\cos(3x)$ has period $\frac{2\pi}{3}$	$2\pi/B$ for each.

If T is a period of f, it is one of $f'' = -4\sin(2x) - 9\cos(3x)$ too	Differentiate $f(x+T) = f(x)$ twice.
So T is a period of $9f + f'' = 5\sin(2x)$ and of $4f + f'' = -5\cos(3x)$	Each combination cancels one term, so each term repeats on its own.
$T = \pi j = \frac{2\pi}{3}k$ for positive integers j, k, so $3j = 2k$; the smallest is $j = 2$, $k = 3$, $T = 2\pi$	A sinusoid repeats only after whole periods: 2π is the least common multiple of π and $\frac{2\pi}{3}$, and $f(x + 2\pi) = f(x)$.
For g: a period would need $T = 2\pi j = \frac{2\pi}{\sqrt{2}}k$, so $\sqrt{2} = k/j$	The same argument, with $g'' = -\sin x - 2\sin(\sqrt{2}\,x)$, makes T a period of each term; but $\sqrt{2}$ is irrational, so no such j, k exist.
f has period 2π; g is not periodic	A sum of sinusoids is periodic exactly when the ratio of their frequencies is rational, and its period is the least common multiple of theirs. g comes close to repeating, but never exactly.

Problem 9

> Start from $y = \sin x$. (a) Stretch the graph horizontally by a factor of 4, then shift it π to the right. (b) Shift it π to the right, then stretch it horizontally by a factor of 4. Write each equation and give its phase shift.

A horizontal stretch by 4 replaces x with $\frac{x}{4}$; a shift right by π replaces x with $x - \pi$	Each moves the point that used to be at x.
(a) Stretch: $\sin\frac{x}{4}$. Then shift: $\sin\frac{x-\pi}{4}$	The shift replaces x in the current equation.
(b) Shift: $\sin(x - \pi)$. Then stretch: $\sin\left(\frac{x}{4} - \pi\right) = \sin\left(\frac{1}{4}(x - 4\pi)\right)$	The stretch also stretches the shift that was already applied.

(a) $y = \sin\left(\frac{1}{4}(x - \pi)\right)$, **phase shift** π; **(b)** $y = \sin\left(\frac{1}{4}(x - 4\pi)\right)$, **phase shift** 4π; **both have period** 8π	The order matters for the shift but not the period. In $A\sin(B(x - C)) + D$ the C is the shift applied last, after the stretch, which is why it is read with B factored out.

Problem 10

A transformer's sinusoidal positional encoding gives position p the features

$$\begin{aligned} \mathrm{PE}(p, 2i) &= \sin(\omega_i p), \\ \mathrm{PE}(p, 2i + 1) &= \cos(\omega_i p), \end{aligned}$$

with $\omega_i = 10000^{-2i/d}$, for $i = 0, \ldots, d/2 - 1$. With $d = 512$, find the period in positions of pair $i = 0$, $i = 128$ and $i = 255$. Then show that for each pair there is a 2×2 matrix R_k, depending on k but not on p, that sends the pair at position p to the pair at position $p + k$.

Pair i has period $2\pi/\omega_i = 2\pi \cdot 10000^{2i/d}$	Period $= 2\pi/B$ with $B = \omega_i$.
$i = 0$: $2\pi \approx 6.28$; $i = 128$: $2\pi \cdot 10000^{1/2} = 200\pi \approx 628.3$	$2i/d = 0$ and $256/512 = \frac{1}{2}$.
$i = 255$: $2\pi \cdot 10000^{510/512} \approx 2\pi \cdot 9646.6 \approx 60611$	The periods grow geometrically from 2π to almost 20000π.
With $a = \omega_i p$ and $b = \omega_i k$: $\sin(a + b) = \cos b \, \sin a + \sin b \, \cos a$	The sine sum formula.
$\cos(a + b) = -\sin b \, \sin a + \cos b \, \cos a$	The cosine sum formula.
Periods $2\pi \approx 6.28$, $200\pi \approx 628.3$ **and about** 60611 **positions;** $R_k = \begin{bmatrix} \cos \omega_i k & \sin \omega_i k \\ -\sin \omega_i k & \cos \omega_i k \end{bmatrix}$ **sends** $(\sin \omega_i p, \cos \omega_i p)$ **to** $(\sin \omega_i (p + k), \cos \omega_i (p + k))$	Steps 4 and 5 are the two rows of R_k applied to $(\sin a, \cos a)$, and only $b = \omega_i k$ appears in R_k. It is a rotation, so an offset of k positions is the same linear map wherever it happens. Fast pairs separate nearby positions; slow pairs tell far-apart positions apart.

Where this goes wrong

1. Reading the phase shift before factoring out B

The argument $2x - \pi/3$ shows a $\pi/3$ in plain sight, and it is tempting to read it as the shift.

$y = 2\sin(2x - \frac{\pi}{3}) + 1$, amplitude 2, period π	Right so far: Problem 2.
"$\sin(x - C)$ is shifted by C, so this is shifted by $\frac{\pi}{3}$."	The habit that causes the mistake: the rule holds only when the coefficient of x is 1.
Phase shift $\frac{\pi}{3}$, first maximum at $\frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12}$ ✗	At $x = \frac{7\pi}{12}$ the argument is $\frac{7\pi}{6} - \frac{\pi}{3} = \frac{5\pi}{6}$, so $y = 2 \cdot \frac{1}{2} + 1 = 2$, not the maximum 3. Factoring gives $2(x - \frac{\pi}{6})$: the shift is $\frac{\pi}{6}$ and the peak is at $\frac{5\pi}{12}$.

2. Taking the period as 2πB

Larger B means more of something, and the period is the quantity it is easy to attach that to.

$y = 3\sin(4x) - 1$, $B = 4$	Right so far: Problem 1.
"The period of $\sin x$ is 2π, and B scales it."	The slip: B scales the speed of the cycle, so it divides the period.
Period $2\pi \cdot 4 = 8\pi$ ✗	The graph completes a cycle when $4x$ grows by 2π, that is when x grows by $\frac{\pi}{2}$. The wrong period is 16 times too long, and a sketch with it shows one bump where there should be sixteen.

3. Taking the maximum as the amplitude

The amplitude is the "height" of a wave, and the maximum is the height that shows on a graph.

Maximum 7 at $x = 1$, next minimum -1 at $x = 4$	Right so far: Problem 5.

“The amplitude is how high the wave goes: 7.”	The habit that causes the mistake: the amplitude is measured from the midline, not from $y = 0$.
$A = 7$, $D = 3$ ✗	This curve reaches $3 + 7 = 10$ and $3 - 7 = -4$, neither of which the data shows. The amplitude is $\frac{7-(-1)}{2} = 4$. Taking the maximum works only when the midline is $y = 0$.

4. Dropping the midline's sign

The vertical shift is written last, and its sign is read the way the phase shift's is.

$y = 3\sin(4x) - 1$, amplitude 3	Right so far: Problem 1.
“A vertical shift written -1 moves the graph by 1, the same way $x - C$ moves it by C.”	The analogy that causes the mistake: inside the argument a minus sign means a move in the positive direction, but outside it means what it says.
Midline $y = 1$, range $[-2, 4]$ ✗	At $x = \frac{\pi}{8}$, where $\sin(4x) = 1$, the function is $3 - 1 = 2$, not 4. The midline is $y = -1$ and the range $[-4, 2]$.

5. Evaluating in degrees

Calculators keep the last mode they were in, and a daylight model is a natural place to think in degrees.

$H(172) = 12 + 3\sin(1.5837)$	Right so far: Problem 6, step 2. The 2π in the formula means the argument is in radians.
“Type $\sin(1.5837)$ into the calculator.”	The slip: the calculator is in degree mode.
$H(172) \approx 12 + 3(0.0276) \approx 12.08$ hours ✗	That is 1.5837 degrees, not radians. In radians $\sin(1.5837) \approx 0.99992$ and $H(172) \approx 15.00$. A result near the midline on the longest day of the year is the warning sign.

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